

EXPERT WITNESS REPORT OF MARK C. NICELY, MATHEMATICIAN
AND CASINO GAMING CONSULTANT

I, Mark C. Nicely, hereby provide the following expert witness report on behalf of
Petitioners, [REDACTED]

I. QUALIFICATIONS.

I am trained as a mathematician and computer engineer, having graduated with a Bachelors of Science degree in Electrical and Computer Systems Engineering from Rensselaer Polytechnic Institute. I am currently a director of gaming design for a game division of International Game Technology (IGT), the world's largest casino slot game and casino systems manufacturing company. I have extensive experience in the casino and gaming industry, as reflected in my curriculum vitae attached as Exhibit 1.

II. ENGAGEMENT AND SUMMARY OF OPINIONS.

I have been retained by Petitioners [REDACTED] through their counsel, to (A) explain the data listed in the Acres Advantages Patron Management report from [REDACTED] Casino [REDACTED] attached as Exhibit 2 and (B) calculate and explain the odds that the Petitioners could have won more than their total gambling losses for a given year from any amount of untracked gambling (i.e. from profits made solely from machine-paid awards whilst playing without a Player Club Card.)

It is my opinion, as discussed more fully below, that for the years 2004 and 2005, the odds are astronomically unlikely that the Petitioners achieved overall net winnings from gambling in either year.

III. DISCUSSION OF PLAYER TRACKING REPORT.

A. General Description of the Player Tracking.

One of the primary purposes of a player tracking system such as the Acres Advantage System, is to collect, store and report data captured from any game transaction on a slot machine or video poker machine when a Player Club Card is active. When a player inserts his/her Player Club Card into a given machine, there is usually one or more visual indicators to the player as to whether the card is active. Most player tracking devices have an LED or LCD display that can display messages to the player, such as:

Insert Player Card

Re-Insert Player Card

**Card Accepted – Welcome <player name>
You currently have XXXX points**

Most player tracking devices also have a lighted boarder or bezel surrounding the Player Club Card insertion slot. The bezel displays a red color if no Player Club Card is inserted or if an inserted Player Club Card is not properly read. If a Player Club Card is inserted and has been properly read, the bezel displays a green color. Just as one may have to reswipe a credit card one or more times in order for the credit card to be correctly read by a credit card reader, a given slot machine Player Club Card reader may require a player to reinsert his/her Player Club Card one or more additional times until said Player Club Card has been properly read.

When a player's Player Club Card has been correctly read, every game subsequently played while that Player Club Card remains in the Player Club Card reader is associated with that player's player tracking account. When a Player Club Card is removed from a Player Club Card reader, the Player Club Card is unregistered from that machine.

If a player does not insert his/her Player Club Card into a given machine or if the machine does not correctly read the Player Club Card and the player does not reinsert the Player Club Card until a valid card read has occurred, then any play on that machine by that player will not be associated with that player's Player Club Card account.

The primary reason most casinos purchase and operate player tracking systems such as the Acres Advantage System is to support a player loyalty program wherein a player's tracked machine play (i.e. gambling on a slot machine or video poker machine whilst using his/her Player Club Card) can earn him/her possible perquisites such as premium parking, shorter VIP lines for buffet, special room rates, special newsletter, invitations to special events such as slot tournament, gifts such as show tickets, shirts, hats, etc., entries into special drawings, free play, and/or other rewards. In order to accurately and fairly ascertain what reward a given player should be offered, a player tracking system is used to record actual play. Relative to the player loyalty program rules established by a given casino, a player earns another loyalty point for every n dollars of play on a certain type of machine. For example, at some casinos, a player earns 1 loyalty point for every \$1 wagered during player-tracked play on slot game and earns 1 point for every \$2 wagered during player-tracked play on a video poker machine.

Many player tracking systems can and do track estimated or actual player table game wagers. Since no player tracking information for table games is provided in the report in Exhibit 2, I will not discuss the corresponding table game player tracking operational details, which are different and more complex than with the tracking of slot machine and video poker machine play.

Player tracking systems can also be used for optimizing casino operations such as determining the effectiveness of mailers, promotions and other marketing campaigns, understanding when, how and where higher value players play, etc.

A non-gambling equivalent of a player tracking system would be a frequent flier program offered by most airlines. A given airline customer gets loyalty program credit for each flight completed with a valid frequent flier number just like a given casino player gets loyalty program credit for each play session completed with a valid Player Club Card. Conventional wisdom indicates that airline frequent flier tracking systems are basically accurate, excluding occasional problems due to error made during hand-entry of a frequent flier number by a customer or agent. While casino player tracking systems of slot machine play have the same inherent systemic accuracy as airline frequent flier systems, they enjoy even higher effective accuracy as there are not equivalent occasions for humans to enter in player club numbers by hand, nor are there complexities such as refunded or upgraded tickets, partially completed itineraries, etc.

Specifically, I have never heard of any problems or read of any accuracy issues over the years regarding player tracking of slot machine and video poker machine play in any of the major industry trade press to which I subscribe (Casino Slot Player, Strictly Slots, International Gaming and Wagering Business). [Note: there have been numerous press articles regarding player disputes in the area of table game tracking when casino staff estimation is required to perform the tracking, but such issues are not germane to the issue at hand.]

Frequent flier programs and player tracking programs are widely known to be accurate because there are so many users (be it fliers or gamblers) continually checking the accuracy of their loyalty point totals; any systemic problem in any such a system would be exceedingly unlikely to go unnoticed and unreported.

B. Slot Statistics Entries on the Acres Advantage Patron Management Report.

The player tracking report included in Exhibit 2 is a report that has the title “Acres Advantage Patron Management – [Player Workbook –]”. This report is consistent with the output of an Acres Advantage System. The Acres Advantage System was developed and sold by Acres Gaming, Inc. Acres Gaming, Inc. was acquired in 2003, by the company for which I presently work: International Game Technology (IGT). IGT has taken over the product support responsibilities for Acres Advantage Systems, including the specific Acres Advantage Systems that generated the report in Exhibit 2.

Definition of terms used in Slot Statistics

The value of each is expressed in US dollars, for the identified player, for the listed period of time:

Coin In: Total actual wagers placed

Coin Out: Total actual winnings paid by machine

Jackpots: Total actual winnings paid by hand, such as when W-2G is required

Act Win: Actual casino profit (loss)

Theo Win: Average mathematically expected casino profit relative to actual wagers

Free Cash Play Used: The nominal wager value of free game play provided to the player by the casino

Additional information:

The phrase “wagers placed” means money that is irreversibly committed to the casino. For example, when I insert a \$20 bill into a slot machine or a video poker machine in the United States, this is always a reversible action. I am not committed to having to bet all or even some of

this money. I can change my mind, press a CASH OUT button and receive the equivalent of my \$20 back (though it may be in a different form, such as in coins, tokens, redeemable voucher). Only when I activate a gambling proposition for a specific amount of money, for example, one spin of the reels in a slot machine for \$5, have I irreversibly placed a wager.

The phrase “paid by hand” means that a player receives payment from a casino staff member, in lieu of the slot machine or video poker machine having done so. An award paid by hand (also known as a “hand pay”) could involve any of the forms of money that a machine might dispense (as listed above), and/or could involve a form of money which typically comes only from human action, such as a check, cashier’s check, money-order, etc.

However, the process of a player redeeming tokens, vouchers, etc. for cash money is not considered to be, and is not referred to in casino parlance as, a “paid by hand” or “hand pay” event.

Slot machines and video poker machines typically have a programmable setting for the award amount threshold for triggering a hand pay. For machines to be placed in the United States, the threshold amount is set at the factory to be \$1,200, consistent with the W-2G threshold. This assures that the required W-2G award procedures and paperwork are properly completed before the corresponding winnings are given to the player.

On many slot machines and video poker machines, the operator is able to change the hand-pay threshold settings. Though I am aware of some operators that have lowered the jackpot-pay threshold setting to an amount lower than \$1,200, I have only seen this done for some coin-based, low-denomination, low-stakes machines. A prime example of such a device would be coin-dispensing, electro-mechanical, nickel denominated slot machines with a max bet of either 15 cents (three nickel bet on the single available payline) or 25 cents (one nickel bet on

each of five available paylines) which were relatively prevalent in Nevada, Atlantic City and Mississippi until they started to be displaced by multi-coin/multi-line video-based slot machines starting around 10 years ago.

The main reasons why some casinos used to set such low-denomination, low-stake, coin-dispensing machines, discussed next, are not really applicable to higher denomination devices (\$1 or higher), or to high-stake devices (on the order of \$1 or higher), and are certainly not applicable to Ticket-In / Ticket-Out devices which dispense only vouchers and which neither accept nor dispense coins.

One main reason that a casino might lower the jackpot-pay threshold on a coin-based, low-stake, nickel machine relates to hopper fills. Before the Ticket-In/Ticket-Out technology, which prevails on present-day casino floors in the United States, slot machines accepted coins as well as notes and dispensed coins. The coins available for dispensing sit in a storage area known as a hopper. If the required number of coins are not available to completely fulfill a player's cash out request, the machine locks up and displays a message such as "CALL ATTENDANT". For the player's cash out request to complete, a casino staff member (usually two staff members, based upon the given casino's security policy) must open the machine and refill the hopper with coins. This typically takes quite a few minutes to accomplish *after* the appropriate casino personnel arrive, which in itself can take an indefinite amount of time. In my experience, trying to get the attention of casino floor staff, in a noisy environment, without going so far from my machine that I risk some other patron trying to sit at my machine, is extremely frustrating.

Some casinos used to set the jackpot threshold on a coin-based, low-stake nickel machine at a sub W-2G threshold, such as \$500 or \$250, for the following reason. Were the player to try to immediately cash out a \$500 win, this would require 10,000 coins and this would most

certainly cause a hopper-fill machine lock-up event with all of the corresponding negative customer consequences. Even if the player continues playing on the same machine, with a maximum wagering of 15 cents or 25 cents per play, he/she is unlikely to whittle down his/her balance significantly. By setting the jackpot threshold to the top award value for a low-stake nickel machine, the casino can exchange a certain-to-be annoying hopper-fill hand-pay experience with a jackpot-win hand-pay experience. The latter is expected to be more pleasant for the player for a few reasons. A jackpot-win lock up is typically accompanied by a rather loud jackpot bell and flashing lights. The player can “bask in fame” as the clearly identified big winner amongst nearby patrons. (This speaks to the other reason that casinos will do this – to create an environment of excitement. It is conceivable that a casino might set a \$1 denominated, \$2 max bet, enormous Big-Bertha style device with a low jackpot-pay threshold for the spectacle it creates.) When a player cashes out, he/she was intending to leave that machine, possibly for some other activity. However, a player encountering a jackpot-win lock-up was likely not planning on leaving, so remaining at the machine is not as drastic of a change of plans. The audio/visual commotion is likely to lead to the arrival of casino staff personnel much more quickly than in a hopper-fill situation and likewise can be resolved much faster: it takes much less time to pay the player \$500 in notes and reset the machine (usually without having to open the machine), relative to the significant amount of time needed to refill a hopper.

However, I have never heard of a casino setting a sub \$1,200 jackpot-pay threshold on a high-denomination (\$1 and above) slot machine, or high-stake (\$1 to \$2 or higher) slot machine. High denominated devices, such as denominations of \$1, \$5, \$25 and \$100 are often specially designed to offer awards such as \$1,000 or \$1,199 to expressly allow players to enjoy big wins without having to suffer through a W-2G lock-up. Indeed, the Petitioner,

confirms that she never experienced a machine lock-up for an award less than \$1,200. Therefore, any values listed in the Jackpot column in Exhibit 2, I recognize as coming from a W-2G recorded award.

The interrelationship of the first four primary statistics can be described as follows:

$$(1) \text{ Coin In} = \text{Coin Out} + \text{Jackpots} + \text{Act Win}$$

Or conversely:

$$(2) \text{ Coin In} - (\text{Coin Out} + \text{Jackpots}) = \text{Act Win}$$

This can also be expressed in a more verbal fashion:

Casino profit is total player wagers minus awards paid to player. Awards paid to player include awards directly paid by the slot machine as well as awards handed to the player by casino staff.

The above listed terms are all well-known and much-used terms within the gambling industry. For example, the terms “Coin In”, “Coin Out” and “Jackpots” can all be found throughout the Nevada’s Gaming Statutes and Regulation, such as the following section of the regulations from the Nevada Gaming Commission and State Control Board Web site:

http://gaming.nv.gov/stats_regs.htm. Please note: I have highlighted and underlined the terms—these highlights and underlines are not in the original.

2.040 Meters for conventional gaming devices, system supported and system based games.

1. All gaming devices must be equipped with electronic digital storage meters of at least 10 digits capable of displaying the information listed in this section on demand. These meters, listed below, must accumulate the following information in units equal to the denomination of the device or in dollars and cents. Devices configured for multi-denomination play must display the required information in dollars and cents.

(a) **Coin In**. The machine must have a meter specifically labeled “Coin In” that accumulates the total value of all wagers, whether the wagered amount results from the insertion of coins, tokens, currency, deduction from a credit meter or any other means. This meter shall:

- (1) Not include subsequent wagers of intermediate winnings accumulated during game play sequence such as those acquired from “double up” games;
- (2) For multi-game and multi-denomination/multi-game gaming devices, provide the information necessary, on a per payable basis, to calculate a weighted average theoretical payback percentage; and
- (3) For gaming devices which are considered slot machines and which contain paytables with a difference in theoretical payback percentage which exceeds 4 percent between wager categories, maintain and display coin in meters and the associated theoretical payback percentage, for each wager category with a different theoretical payback percentage, and calculate a weighted average theoretical payback percentage for that payable;
- (b) **Coin Out**. The machine must have a meter specifically labeled “Coin Out” that accumulates the total value of all amounts directly paid by the machine as a result of winning wagers, whether the payout is made from the hopper, to a credit meter or by any other means. This meter will not record amounts awarded as the result of an external bonusing system or a progressive payout;
- (c) Coin Drop. The machine must
- (c) Coin Drop. The machine must have a meter specifically labeled “Coin Drop” that accumulates the total value of coins or tokens diverted to the drop;
- (d) Attendant Paid **Jackpots**. The machine must have a meter specifically labeled “Attendant Paid Jackpots” that accumulates the total value of credits paid by an attendant resulting from a single winning alignment or combination, the amount of which is not capable of being paid by the machine itself. This does not include progressive amounts or amounts awarded as a result of an external bonusing system. This meter is only to include awards resulting from a specifically identified amount listed in the manufacturer’s par sheet;

C. Slot Statistics Example.

The following example shows how these terms apply to real-world events.

- ◆ I insert a \$50 bill into a \$1 denomination slot machine
- ◆ The slot machine shows that I have a balance of 50 credits
- ◆ At this point, since I have not committed a bet:
 - Coin In, Coin Out, Jackpots and Act Win are all \$0
- ◆ I commit a \$1 bet by pressing the PLAY button
- ◆ 1 credit = 1 dollar is deducted from my credit balance (50 credits → 49 credits)
- ◆ I get a losing outcome (e.g. Blank-Seven-1Bar)
- ◆ At this point, my slot statistics are:
 - Coin In = \$1
 - Coin Out = \$0
 - Jackpots = \$0

- Act Win = \$1 (i.e. the casino won a dollar and I lost a dollar)
- ◆ I commit another \$1 bet by pressing the PLAY button
- ◆ 1 credit = 1 dollar is deducted from my credit balance (49 credits → 48 credits)
- ◆ I get a winning outcome that pays 5x my bet (e.g. Cherry-Cherry-Blank)
- ◆ At this point, my primary slot statistics are:
 - Coin In = \$2
 - Coin Out = \$5
 - Jackpots = \$0
 - Act Win = minus \$3 (i.e. for this overall play session, the casino is down \$3 and I am up \$3)
- ◆ Note that my machine-paid award went directly to my credit balance on the machine: 48 credits at the start of the proposition plus 5 credits award results in a new credits balance of 48 credits + 5 credits = 53 credits
- ◆ If I press the CASH OUT button on the machine at this point, I would receive \$53 in some form (coins, token, vouchers, etc.) that I can carry away from the machine. The process of cashing out would not change my primary slot statistics. Namely, I placed \$2 in wagers, received \$5 in awards, and came out ahead \$3 (which of course means a corresponding \$3 loss to the casino).
- ◆ Alternatively, I can continue playing on this same machine and can recirculate award credits awarded by earlier wins into new wagers. For example, with 53 credits in my balance, I have enough money to be able to play at least 53 more times, at \$1 per play, without having to insert any additional money.
- ◆ Let's say, as quite often happens to players in real life, I continue playing until my balance drops to 0 credits. Because I can, and do, recirculate winning credits back into new wagers, my original \$50 buy-in can result in my being able to play over 100

games and maybe over 200 games or more until my balance drops to 0. The more wins I have, the more money I have to bet, the more games I can play without having to insert any additional money into the machine.

◆ In the case where I was able to play 180 games until my balance drops to 0, my primary slot statics would be:

- Coin In = \$180
- Coin Out = \$130
- Jackpots = \$0
- Act Win = \$50

In other words, my original \$50 allowed me to play 180 games.

At \$1 per game, this means I wagered \$180. Of the \$180 of my wagers, \$50 was from my original deposit and \$130 was from awards paid to me from which I elected to make additional bets.

\$50 original buy-in amount + \$130 recirculated awards = \$180 total wagers.

At the end of this play session, I walk away from the machine with \$50 less money in my pocket than when I arrived (which likewise means the casino now has \$50 more money). For this example play session, my net gambling loss is therefore \$50 and, likewise, the casino's win is \$50.

D. Acres Advantage Patron Management Report Specifics.

Looking at the first column of data titled "2004" in Exhibit 2 we see the following data:

Coin In	8,517,492
Coin Out	7,034,435
Jackpots	1,114,645
Act Win	368,413

As discussed above, these data indicate that for [redacted] player-tracked machine play (i.e. gambling while using her Player Tracking Card,) [redacted] made \$8,517,492 in wagers, resulting in \$7,034,435 from machine-paid awards and \$1,114,645 from staff-paid awards, or $\$7,034,435 + \$1,114,645 = \$8,149,080$ of total awards. Therefore, [redacted]'s net [redacted] 2004 player-tracked gambling profit (loss) is easily calculated:

Paid to Casino	\$ (8,517,492)
Won from Casino	\$ 8,149,080
Net Player Gambling Profit (Loss)	\$ (368,412)

and shows that [redacted] suffered a net gambling **loss** of \$368,412 for the year 2004 from [redacted] Casino from player-tracked play. [redacted]'s \$368,412 gambling loss for 2004 correctly corresponds with the [redacted] Casino reporting having won \$368,413 (Act Win) from her for 2004. (The \$1 discrepancy between [redacted]'s \$368,412 gambling loss [redacted] from player-tracked play's corresponding \$368,413 gambling win can be explained by rounding errors from not having the cents portion of the monetary values printed in the report.)

In similar fashion, the first column of data titled "2005" in Exhibit 2 shows

Coin In	2,846,048
Coin Out	2,217,317
Jackpots	499,040
Act Win	129,691

These data indicate that for [redacted] player-tracked machine play (i.e. gambling while using her Player Tracking Card,) [redacted] made \$2,846,048 in wagers,

resulting in \$2,217,317 from machine-paid awards and \$499,040 from staff-paid awards, or $\$2,217,317 + \$499,040 = \$2,716,357$ of total awards.

Therefore, [redacted]'s net [redacted] 2005 gambling profit (loss) is calculated:

Paid to Casino	\$ (2,846,048)
Won from Casino	\$ 2,716,357
Net Player Gambling Profit (Loss)	\$ (129,691)

and shows that [redacted] suffered a net gambling **loss** of \$129,691 for the year 2005 from [redacted] Casino from player-tracked play. [redacted]'s gambling loss for 2005 exactly corresponds with the [redacted] Casino reporting having won \$129,691 (Act Win) from her for 2005.

E. Summary.

Based on the Acres System Patron Management Report shown in Exhibit 2, [redacted] [redacted] suffered net gambling losses of \$368,412 and \$129,691 from player-tracked gaming machine play at [redacted] Casino [redacted] for the years 2004 and 2005, respectively.

IV. DISCUSSION OF POTENTIAL WINNINGS FROM UNTRACKED SLOT PLAY.

A. Purpose.

In this section, I calculate the odds that the Petitioner could have attained a given amount of untracked profit, that is, profits made solely from machine-paid awards whilst playing without a Player Club Card. The analysis is limited to machine-paid awards, thereby excluding staff-paid awards, since such staff-paid awards would be in conjunction with W-2G winnings, which are already accounted for.

B. Overall Approach.

I use an industry-standard confidence interval test to solve the stated problem. I obtain precise inputs relating to slot machine payout behavior for this test by analyzing the data available in Exhibit 2. I do not have to rely upon any assumptions or industry-based estimates regarding game payout as I am able to directly measure a very large sample of play from the Exhibit 2 data.

The value that I do not know, which is n , the number of untracked games played, I examine as a variable, not a constant. Specifically, I obtain a result which indicates the required probability for any number of games played without a Player Club Card. Furthermore, I specifically a) identify the number of games most likely to produce the lowest test metric within this scenario most likely to attain the required break-even profit and b) calculate the corresponding probability. This covers the Petitioner's worse-case scenario.

C. Analysis Approach.

The basic equation that applies in such a situation is the confidence interval equation:

$$(3) \Delta = \frac{z * sd}{\sqrt{n}}$$

Where:

- ◆ Δ or "delta" is the difference between actual and expected
- ◆ z is the normal distribution z-score which represents the number of standard deviations from the mean our range is
- ◆ sd is the standard deviation
- ◆ n is the number of games played

Equation (3) can be rearranged to calculate a z-score:

$$(4) z = \frac{\Delta * \sqrt{n}}{sd}$$

A given z-score indicates the probability of natural occurrence of the measurement at hand relative to the standard normal distribution equation:

$$(5) \text{ probability of natural occurrence} = f(z) = 1 - \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

A z-score above 3.0 is considered improbable. In most statistics text books, calculated z-score tables usually only go as high as 3.0. A z-score of 6.0 is considered extremely improbable, with odds in excess of 1 billion to 1 against natural occurrence.

To calculate the value for sd required for equation (4), I follow this standard definition equation for weighted standard deviation:

$$(6) \text{ standard deviation} = \sqrt{\text{variance}}$$

$$(7) \text{ variance} = \sum_{i=1}^n [p(i) * (x_i - \bar{x})^2]$$

Where:

- ◆ $p(i)$ is proportion of occurrence of event i
- ◆ x_i is the value of event i
- ◆ \bar{x} is the mean value of the distribution

I apply equations (6) and (7) to the report data from Exhibit 2, as shown in Exhibit 3. The terms used for equation (7) :

- ◆ $p(i)$ = Coin In for Month i / Total Coin In
- ◆ x_i = Actual Base RTP for Month i
- ◆ \bar{x} = Total Theo Base RTP

- ◆ RTP, or Return to Player, represents the average proportion of a player's wagers returned to player in winnings

As shown in Exhibit 3, *sd*, the standard deviation of RTP derived from [redacted]'s actual machine-game records is 3.29%.

The Δ value required to solve equation (4) needs to be calculated relative to the question to be answered. Specifically, my objective is to calculate the odds that [redacted] [redacted] enjoyed sufficient gambling profit from machine-only pays (i.e. awards not paid by hand and recorded on a W-2G form) while not using her Player Club Card, to compensate for the total net gaming losses indicated by the Patron Management report in Exhibit 2. Specifically, what are the odds that in 2005, [redacted] earned \$129,691 or more from machine-only pays when she was not using her Player Club Card?

To achieve a given level of total winnings from machine-only pays, I need to calculate Δ , the difference between target wins and theoretically expected wins, as follows:

$$(8) \Delta = |(\text{RequiredRTP} - \text{ExpectedRTP})|$$

Where RTP (Return To Player) is defined as the ratio (Coin Out) / (Coin In).

The RTP required to generate \$129K of profit from a given Coin In can be expressed as:

$$(9) \text{RequiredRTP} = (\text{TargetProfit} + \text{Coin In}) / \text{Coin In} = (\text{TargetProfit} / \text{Coin In}) + 1$$

Therefore (8) can be expressed as:

$$(10) \Delta = |(\text{TargetProfit} / \text{Coin In}) + 100\% - \text{ExpectedRTP}|$$


By definition, $\text{expectedWin\%} = 100\% - \text{expectedRTP} = \text{the expected return to casino} = \text{expected casino win}$. Specifically, $\text{expectedWin\%} = 100\% - 82.1\% = 17.9\%$. Additionally, the value for total player wagers, Coin In, can be expressed as $n * \text{averageBet}$.

Therefore (10) can be further simplified as

$$(11) \Delta = \frac{\text{TargetProfit}}{n * \text{AverageBet}} + \text{ExpectedWin\%}$$

Note that the absolute value function in (8) and (10) are no longer required in (11) since all terms are positive numbers, which guarantees that the result will likewise be a positive number.

D. Analysis Calculation.

In order to solve (11), as well as to obtain the value n (the number of games) for equation (4), I need to use a reasonable bet size.  informed me that her typical slot machine wager was between \$10 and \$20 per game, but to be conservative, I assume an even larger average machine-game wager of \$25. This is a conservative choice because the larger the wager is, the easier it is to achieve a certain win threshold within a certain number of plays.

Therefore, by substituting the following values:

- ◆ TargetProfit = \$129,691
- ◆ AverageBet = \$25
- ◆ Expected Hold = $100\% - 82.1\% = 17.9\%$

into equations (11), I calculate Δ as:

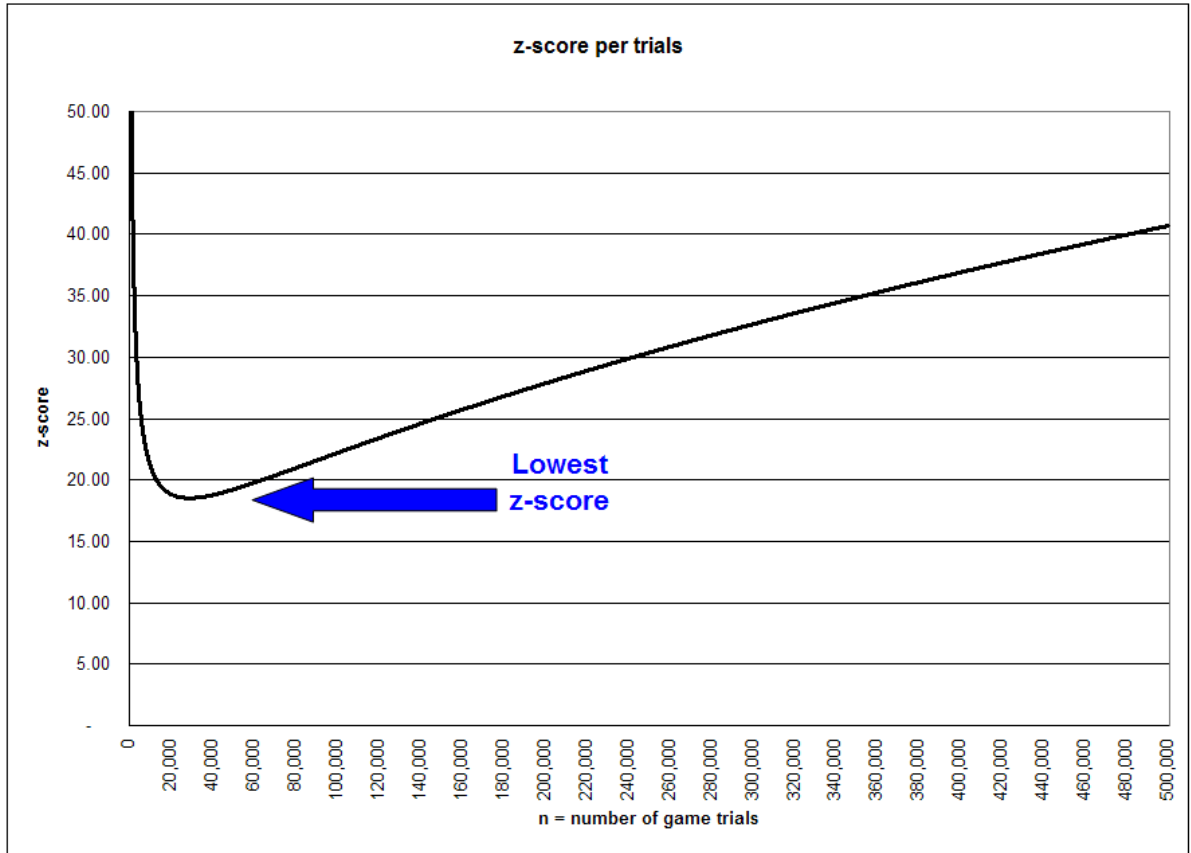
$$(12) \Delta = \frac{\$129,691}{n * \$25} + 17.9\%$$

Substituting this calculation for Δ into equation (4), along with the knowledge that $sd = 3.29\%$, yields:

$$(13) \quad z = \frac{\left(\frac{\$129,691}{n * \$25} + 17.9\%\right) * \sqrt{n}}{3.29\%} = \frac{\left(\frac{5178.64}{n} + 17.9\%\right) * \sqrt{n}}{3.29\%}$$

The absolute minimum number of games in which one could earn \$129,691 or more with awards no larger than \$1,199 is $\left\lceil \frac{\$129,691}{\$1,199} \right\rceil = 109$ games.

By plotting equation (13) for n=109 through n=20,000, we get the following curve (14):



It can be easily seen that the z-score value never drops below 17.5. Using differential calculus¹, the actual minimum z-score is 18.524 which occurs at $n=28,981$. Note that I am not implying the Petitioner actually played these many games without her Player Club Card. What I am saying is that the Petitioner would have had the maximal chance of enjoying profits from sub W-2G winnings had she played these number of games without her Player Club Card.

¹ To find a minima for $z = f(n)$, we differentiate $f(n)$ and solve $f'(n) = 0$. Specifically, we differentiate $f(n) = \left(\frac{1}{3.29\%}\right) \left(\frac{5178.64}{n} + 17.9\%\right) \sqrt{n} = 30.39(5178.64n^{-1} + 17.9\%)n^{1/2} = 157,679n^{-1/2} + 5.44n^{1/2}$

to obtain $f'(n) = (-\frac{1}{2})157,679 * n^{-3/2} + (\frac{1}{2})5.44 * n^{-1/2} = 0$, or $\frac{n}{2}(-157,679 * n + 5.44) = 0$, which has two solutions: $n=0$ is one solution, the other is found by solving $5.44=157,679n$, or $n=5.44/157,679= 28,981.23$.

As discussed earlier, a z-score of 6 is considered extremely improbable – on the order of one chance in a billion. Normal distribution z-score tables typically published in statistics books and mathematical handbooks usually only indicate the probability equivalent of z-score values up to 3.0 (also known as “3 sigma”). One statistics book does list the odds for $z=5$ to be in excess of 1 in 3 million. The NORMSDIST(z) function in the Microsoft Excel 2007 spreadsheet program, does not work for an input z-score value of 7.88 or higher. At $z=7.87$, the Microsoft Excel 2007 NORMSDIST(z) function generates the value 562,949,953,421,312, which, in other words, indicates odds in excess of 1 in 500 trillion. An outcome with a z-score of $z=18.5$ is so astronomically improbable as to be safely considered impossible that the Petitioner could have earned \$129K in machine-paid profits from any amount of non-tracked slot gambling (i.e. from profits made solely from machine-paid awards whilst playing without a Player Club Card).

Obviously, if the odds of earning \$129K in these circumstances are astronomical, then the odds of earning an even larger amount, \$368K in 2004 (which is even more unlikely), are likewise also astronomical.

E. Analysis Summary.

The analysis demonstrates the impossibility of the Petitioner attaining machine-paid gambling winning (i.e. where all awards are below the W-2G reporting threshold) on the order of \$129K or larger when she did not use her Player Club Card, irrespective of the number of games she played without her Player Club Card.

Respectfully Submitted,

A rectangular orange box used to redact the signature of Mark C. Nicely.

MARK C. NICELY

Dated:

A rectangular orange box used to redact the date of the document.

EXHIBIT 3

DATA AND CHART USED FOR CALCULATION OF BASE GAME
STANDARD DEVIATION – *sd*

	RTP						Casino Stats					
	act base	act jackpot	act total	theo total	base theo - base act	variance	Coin In	Coin Out	Jackpot	Act Casino Win	Theo Casino Win	% bet total
Jan-2004	97.7%	0.0%	97.7%	95.5%	15.58%	2.43%	\$129,569	\$126,598	\$0	\$2,972	\$5,819	1.3%
Mar-2004	87.6%	11.1%	98.7%	95.5%	5.47%	0.30%	\$989,671	\$866,987	\$109,564	\$13,120	\$44,577	10.0%
Apr-2004	88.9%	2.5%	91.4%	94.9%	6.76%	0.46%	\$256,119	\$227,668	\$6,379	\$22,072	\$12,960	2.6%
May-2004	82.2%	12.5%	94.7%	95.4%	0.05%	0.00%	\$895,352	\$735,798	\$112,245	\$47,309	\$41,494	9.0%
Jun-2004	80.3%	15.7%	96.0%	95.1%	-1.81%	0.03%	\$1,161,415	\$932,788	\$182,186	\$46,441	\$56,666	11.7%
Jul-2004	84.2%	11.4%	95.6%	94.3%	2.07%	0.04%	\$747,051	\$629,020	\$85,307	\$32,725	\$42,845	7.5%
Aug-2004	81.0%	10.2%	91.2%	95.2%	-1.15%	0.01%	\$525,193	\$425,321	\$53,685	\$46,187	\$24,998	5.3%
Sep-2004	81.9%	16.5%	98.4%	95.4%	-0.27%	0.00%	\$765,638	\$626,737	\$126,598	\$12,303	\$35,242	7.7%
Oct-2004	79.9%	14.4%	94.3%	95.5%	-2.18%	0.05%	\$1,340,760	\$1,071,906	\$193,038	\$75,816	\$60,740	13.5%
Nov-2004	80.5%	8.9%	89.3%	95.3%	-1.64%	0.03%	\$518,295	\$497,669	\$54,727	\$65,900	\$28,959	6.2%
Dec-2004	82.1%	17.5%	99.7%	96.2%	0.00%	0.00%	\$1,088,430	\$893,945	\$190,916	\$3,569	\$41,577	11.0%
Feb-2005	76.6%	18.1%	94.7%	96.0%	-5.52%	0.30%	\$171,713	\$131,546	\$31,134	\$9,032	\$6,859	1.7%
Mar-2005	77.9%	9.1%	87.0%	96.1%	-4.24%	0.18%	\$149,119	\$116,142	\$13,613	\$19,364	\$5,851	1.5%
Apr-2005	81.5%	10.0%	91.5%	95.6%	-0.63%	0.00%	\$326,003	\$265,703	\$32,705	\$27,596	\$14,257	3.3%
May-2005	81.2%	15.4%	96.6%	95.9%	-0.98%	0.01%	\$189,571	\$153,845	\$29,216	\$6,510	\$7,706	1.9%
Aug-2005	80.6%	16.9%	97.6%	96.0%	-1.51%	0.02%	\$289,398	\$233,297	\$49,028	\$7,074	\$11,454	2.9%
Sep-2005	73.3%	19.1%	92.4%	96.3%	-8.85%	0.78%	\$102,024	\$74,760	\$19,524	\$7,740	\$3,729	1.0%
Dec-2005	78.9%	13.7%	92.6%	96.3%	-3.20%	0.10%	\$186,015	\$146,827	\$25,466	\$13,722	\$6,918	1.9%
TOTAL	82.1%	13.2%	95.4%	95.4%	-0.07%	0.11%	\$9,931,336	\$8,156,557	\$1,315,331	\$459,452	\$452,651	100%

Standard Deviation **3.29%**